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# On the evaluation of the moments of inertia of nuclei

**Abstract.** A re-interpretation of previous numerical results based upon the Peierls–Urbano method shows a remarkable agreement of the calculated moment of inertia of $^{28}$Si with the experimental value. In those calculations it is assumed that one knows the true ground-state binding energy. Since there is a possible objection against such a procedure, an alternative way of doing the calculations is proposed.

Recently Peierls and Urbano (1968) have proposed a new method for evaluating the moments of inertia of even–even nuclei which was based upon the Villars theory of nuclear rotation (Villars 1965). However, the result of its application to the solution of a numerical example compared poorly with the experimental data (Peierls and Urbano 1968).

The aim of the present letter is twofold:

(i) We show that the previous result can indeed be made to agree with the empirical data, by a re-interpretation of the experimental level scheme.

(ii) We propose a new way of evaluating the moment of inertia in the framework of the Villars theory, which dispenses with the need for guessing the ground-state binding energy.

The first thing to notice is that the determination of the 'experimental' moment of inertia is not always so clear a procedure as it may seem at first sight. This point is well illustrated by the two-dimensional Villars theory, where the excitation energy of a rotational state of angular momentum $J$ is given, using perturbation theory, by a power series of $J$, the moment of inertia $I$ being defined as to make $1/2I$ the coefficient of the second-order term (Peierls and Urbano 1968). The 'experimental' values of the series coefficients are determined then by fitting the power series for the several values of $J$ to the empirical excitation energies of the states of a given rotational band. If the convergence is good this procedure is well defined, but if the series converges slowly, as may be the case for small bands, some amount of arbitrariness is always involved.

These considerations are pertinent to the example solved previously (Peierls and Urbano 1968), since the $^{28}$Si ground-state band does not follow exactly the ideal rotational pattern. In fact, its only two excited states (a $2+$ and a $4+$ state) have energies (1·78 MeV and 4·61 MeV, respectively (Leder *et al.* 1967)), the ratio of which is 0·386 instead of 0·300 as in the ideal case. There seems to be therefore a case for correcting the usual definition of the experimental moment of inertia, namely

$$E_{2+}-E_{0+}=\frac{\hbar^2}{2I_{\text{exp}}}2(2+1) \tag{1}$$

by including terms of higher order in $J$. It should be noticed, incidentally, that equation (1) leads to

$$\frac{\hbar^2}{2I_{\text{exp}}}=296\,\text{kev}$$

a result which does not compare well enough with the one obtained using the moment of inertia evaluated previously (Peierls and Urbano 1968):

$$\frac{\hbar^2}{2I_{\text{PU}}}=380\,\text{kev}. \tag{2}$$

If one corrects up to the next order only, i.e. if one supposes that the rotational energy is given by

$$E_{J+}-E_{0+}=\frac{\hbar^2}{2I_{\text{exp}}}J(J+1)+CJ^2(J+1)^2 \tag{3}$$

one would obtain

$$\frac{\hbar^2}{2I_{\text{exp}}} = 325 \text{ kev} \tag{4}$$

as the only value which allows equation (3) to reproduce the band energies. The agreement between the theoretical and the experimental results, equations (2) and (4), respectively, is now quite good, the relative errors being well inside the range one should expect owing to the approximations involved in the calculations.

The second part of this letter deals with a possible objection against the method used previously (Peierls and Urbano 1968), namely of its using the true ground-state binding energy $E_0$. This comes about when one evaluates the quantity $F_0$ defined in equation (4.1) of the previous work (Peierls and Urbano 1968) by means of a variational principle to approach the Green function $1/(H^{(0)}-E_0)$, the final value for $F_0$ being written as a function of $E_0$. Lower and upper bounds to $E_0$ are quite difficult to evaluate if the error is to be small, so we propose now to approximate the spectra of the intrinsic energy operators $H^{(0)}$ in the spirit of the usual nuclear theory techniques, for example, the Tamm–Dancoff approximation, R.P.A. approximation, etc.

We have already carried out a Tamm–Dancoff calculation of $F_0$ in the same subspace as that used previously (Peierls and Urbano 1968). We found an 'excited' state with a smaller energy than the 'ground state', but this should not surprise us since as it happens in any kind of Tamm–Dancoff approximation we were looking for the excited states in a space wider than the class of Slater determinants to which the ground-state wave function is meant to belong. Nevertheless, this caused no trouble since it was possible to identify that state with the spurious one $J\mathcal{X}$, $\mathcal{X}$ being the ground-state Slater determinant. Notice that the two states would appear exactly with the same energy if a R.P.A. approximation was used instead. The final result for the moment of inertia led to

$$\frac{\hbar^2}{2I_1} = 345 \text{ kev} \tag{5}$$

which agrees quite well with the 'experimental' 325 kev. We have repeated the calculations, but this time eliminating the $J\mathcal{X}$ state from the very beginning, i.e. diagonalizing $H^{(0)}$ in that part of the subspace which is orthogonal to $J\mathcal{X}$. The final result was

$$\frac{\hbar^2}{2I_2} = 359 \text{ kev}$$

which does not differ much from equation (5).

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